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Group Velocity Effect on Wakefield Calculation

C. Jing

In [1], the author pointed out that the amplitude of the fundamental-mode longitudinal electric wake field for a point source is given by

$$E_z = \frac{q\omega}{2(1-\beta_g)}\frac{R}{Q} \qquad (1),$$

where q is the bunch charge, ω is the wake frequency, R is the shunt impedance, Q is the structure quality factor, and $\beta_g = v_g/c$ is the normalized group velocity. The factor $(1-\beta_g)$ in the denominator of equation (1) is often ignored in linacs having very low group velocity. However, in some cases, group velocity is low enough to be ignored so that we must put this correction factor in for obtaining wake field. In this note we will present the process to get equation (1).

We know, by conservation of energy, the stored energy per unit length must equal the energy loss by preceding charge minus the energy flow out of the volume due to the propagating wakefield itself [2]. Here we define energy flow out with negative value. Then we have

$$U = \Delta U + \frac{P}{c} \qquad (2),$$

where, U is stored energy per unit length, ΔU is particle energy loss, P is power flow out of the volume and c is speed of light. Because power flow is related to stored energy by group velocity v_g, i.e. $U = P/v_g$, we are able to rewrite equation (2) as

$$\Delta U = (1-\beta_g)U \qquad (3).$$

As we know, the stored energy per unit length also can be presented as

$$U = \frac{1}{2}qE_z \qquad (4),$$

And in wakefield calculation, the particle energy loss can be expressed as [3]

$$\Delta U = k_l q^2 \qquad (5),$$

where k_l is loss factor which can be solved by[3]

$$k_l = \frac{1}{4}\omega\frac{R}{Q} \qquad (6).$$

Actually, k_l, ω and R over Q have to be found for each mode of wakefield. Here, for simplification, we just focus on fundamental mode.

Now, we start simple derivation. Firstly, we combine equation (3) and (5), and then we have

$$U = \frac{k_l q^2}{1-\beta_g} \qquad (7).$$

By applying this equation into equation (4) we obtain

$$E_z = \frac{2U}{q} = \frac{2k_l q}{1-\beta_g} \qquad (8).$$

Finally, we put equation (6) in, and it ends up with

$$E_z = \frac{1}{2}\frac{R}{Q}\frac{q\omega}{1-\beta_g} \qquad (9).$$

Or, we can easily calculate R over Q by rewrite equation (9) as

$$\frac{R}{Q} = \frac{2E_z(1-\beta_g)}{q\omega} \qquad (10)$$

if we know wake field by other methods[4].

[1] E. Chojnacki, et al, PAC 1993, pp.815-817
[2] J. G. Power, W. Gai and P. Schoessow, PRE **60,** 1999, pp.6061-6067
[3] D. H. Whittum, SLAC-PUB-7802, unpublished, pp.52-53
[4] M. Rosing and W. Gai, PRD **42,** 1990, pp1829-1834